849

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Discussion of
"THE OCTAGONAL GIRDER FOUR COLUMN SPACE FRAME"

by P. C. Disario, J. S. Podolan, and N. A. Weil
(Proc. Paper 542)

P. C. DISARIO,[1] A. M. ASCE, J. S. PODOLAN,[2] AND N. A. WEIL,[3] ASCE. The authors wish to thank the discussers of their paper for the interest shown and feel that each has contributed to the analysis by his comments.

Mr. Diwan has shown a masterful understanding of the problem and is correct in pointing out that an error exists in equation (26). A recheck of this equation indicates it should read as follows:

$$\text{Revised (26)} \quad \xi_3 = \frac{0.08579K^2 + 0.6395K + 0.5430}{K + 1}$$

Subsequent equations in which this term is employed will require modification.

We agree with Mr. Diwan that the solution given is complex, however included in the paper is a "modified analysis" which produces substantially the same results with considerably less effort. This solution is employed in the actual design of these frames and with the aid of curves makes the design reasonably simple.

Professor Shore and Mr. Rincon have in their discussion questioned the authors' statement that no previous study of the subject of this paper exists. We believe this statement continues to be substantially correct. The authors have attempted to produce coefficients of closed form which can be adapted to the curve type of presentation for rapid solution of design problems.

Professor Shore and Mr. Rincon have used the obvious method of moment distribution to arrive at moments for uniformly distributed vertical load; based upon this method they state that "horizontal shear loading and moment loading will not present any difficulties." It is unfortunate that Professor Shore and Mr. Rincon do not attempt to prove this point more thoroughly, for it is the authors' belief that the method of moment distribution when applied to the octagonal girder space frame for horizontal and moment loading will present difficulties not normally encountered in usual design problems.

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Discussion of
"PRESTRESSING PRACTICE IN BRIDGE BUILDING"

by J. C. Rundlett

J. J. POLIVKA,* M. ASCE.—The paper is a very valuable review of bridges in prestressed concrete built in the U. S. and of methods used for post- and pretensioning of high-tensile cables and wires. The author points also to the advantages of the Hoyer's method of pretensioning which has been adopted by the progressive Concrete Products Company in Pennsylvania, consisting basically in stretching wires or strands on long casting beds. It is fair to give credit to the originator of this highly economical method, to Carl Wettstein, as it has been done previously in several American publications.[1] The writer likes to reproduce Wettstein's statement in this respect which has been confirmed by Curzon Dobell, Paul W. Abeles and other specialists in prestressed concrete: "It can be understood why Hoyer often is considered as inventor of concrete with pretensioned wires. Hoyer was interested in acquiring the license of my process and visited me several times during the years 1927-1931. He got acquainted with all pertinent details in my factory in Most, Bohemia, especially gages and quality of wires, intensity of prestressing, vibration methods, curing, hardening, cutting, etc. Loading tests were made in his presence, and all details of manufacturing were revealed to him. Under Nazi Germany his manufacturing plant was largely subsidized by the government, with contributions of over 4 million marks. He never referred to my process and patents (first patent granted to Wettstein in 1921), and finally, with the support of the Nazi party he was granted his infringing patent. . . My pretensioned concrete units manufactured and used in construction since 1919 are not only a pioneer work in this field, but they still are products of the highest perfection." During the Nazi occupation of Czechoslovakia, 1939-'45, Wettstein had to leave his country, and after the second world war factories for his products were built in South America, Germany and other countries. The first factory in the U. S. was organized and built in New Jersey, 1949, under the personal supervision of Wettstein and his son.[2]

The paper is a stimulating contribution to greater use of reinforced concrete for bridge construction and further progress. The American engineers are credited with having designed and built the longest spans of steel bridges (Golden Gate bridge in San Francisco, 4200 ft.; Messina Straits Bridge designed by D. B. Steinman 5000 ft.) and should be leading also in concrete bridges. The longest concrete arch span in the United States (George Westinghouse Bridge in East Pittsburgh)[3] is only about half of the Sandoe Bridge in Sweden. The maximum span of the prestressed Walnut Lane Bridge in Philadelphia is less than half of the Moselle River Bridge at Coblenz, Germany (403.7 ft).[4] Also the economy of prestressed concrete bridges is not to be overlooked. According to John C. Rundlett's reports (published in other technical periodicals) the cost of the superstructure for spans up to about 100 ft., averages approx. $10.00 per sq. ft.

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* Cons. Engr., Berkeley, Calif.; Lecturer, Stanford Univ., Stanford, Calif.

1. Curzon Dobell, Patents and Codes Relating to Prestressed Concrete, ACI Journal, May 1950, Proc. V. 46, p. 713; Abeles, The Principles and Practice of Prestressed Concrete, New York, 1950, Frederick Ungar Publishing Co.
2. Engineering News Record, Oct. 20, 1949, p. 100.

NOTE: Footnotes 3 and 4 appear at the top of the following page.

3. J. J. Polivka, Contractor Meets Close Design Tolerances in Building Long-Span Concrete Arch Bridge," Civil Engineering, Jan. 1949, p. 40; also: "Proposed Construction Methods on San Francisco Butterfly Bridge," Pacific Road Builder and Engineering Review, June, 1953.
4. A. E. Komendant, "Concrete Bridge Erected by Cantilever Method," Engineering News-Record, Jan. 27, 1955, p. 77. See also complete analysis of a continuous prestressed concrete bridge with 400 ft. center span in Komendant's book "Prestressed Concrete Structures," pp. 108-124, McGraw-Hill, 1952.

ARTHUR R. ANDERSON,[1] A.M. ASCE.—Mr. Rundlett's compilation of contemporary design in American prestressed concrete bridges is an excellent resumé of progress made during the first five years after the Walnut Lane Bridge in Philadelphia.

This writer must concur with the author in the matter of general lag in recognition for the structural possibilities of prestressing.

However, within the past two years his firm has developed and fabricated a number of county road bridges in the State of Washington.

In the main, these bridges consist of a deck section containing six modified T girders with a top flange 48 inches wide, and a total depth of 30 inches for spans up to 60 feet, and a depth of 42 inches for spans between 60 and 90 feet, as illustrated by the accompanying figures 1 and 2.

The girders are pretensioned with 7-wire high tensile strength strands, stressed initially on the line to 200,000 psi. The tension is transferred to the concrete after it reaches a compression strength of at least 5000 psi, at which time a good bond anchorage is assured.

Two types of concrete have been used:

8 sack - sand - gravel aggregate mix,
8 sack - sand - haydite aggregate mix.

The gravel aggregate concrete weighs 155 lbs. per cubic foot, and develops a 28-day compression strength in excess of 7500 psi, usually reaching more than 10,000 psi.

The Haydite concrete weighs 115 lbs. per cubic foot and develops a 28-day compression strength in excess of 6000 psi, usually reaching 7000 psi.

The girders are delivered from the factory to the site by truck and trailer. When the girders exceed 60 feet in length, a steering trailer is used. The driver of the steering trailer communicates with the truck driver by telephone. Figure 3 illustrated the truck and steering trailer delivering a bridge girder 80 feet long, to a site 125 miles from the factory.

Transverse diaphragms cast to the girder are located at intervals of 15 to 20 feet. The diaphragms are provided with pipe sleeves, through which transverse tendons are post-tensioned after the girders have been erected and grouted. The top surface of the girders becomes the pavement, and no further wearing surface is required.

The Rainbow Falls State Park Bridge, erected in 1954, consists of three spans, of 40 - 60 and 40 feet. The middle span crosses a river the depth of which is 70 feet. The girders were erected with a sky-line of two-inch cable rigged to a tall tree by a logging expert. The girders were lowered on the piers and abutments from the skyline by chain falls, shown in figure 4. The completed bridge is shown in figure 5.

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1. Partner, Concrete Eng. Co., Tacoma, Wash.

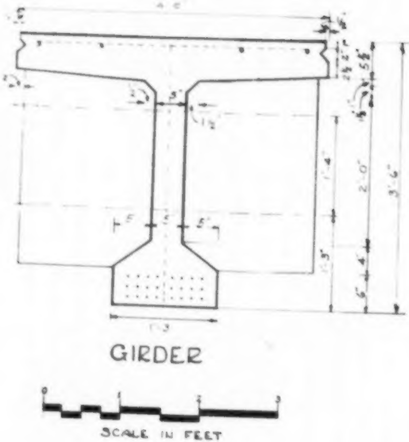
In 1954, Klickitat County, in Washington State, constructed three 91 foot spans and two 47 foot spans with prestressed concrete beams arranged similar to the Willow Creek Bridge in Oregon.

The Washington State Highway Department completed its first prestressed bridge in 1955. Located over the Samamish River about ten miles East of Seattle, this bridge was made up of five spans of 32 feet, each span having twelve pretensioned beams 24" deep. Six prestressed concrete pile bents were used for the substructure.

The City of Ketchikan, Alaska, now has under construction a Viaduct containing 186 pretensioned beams of 42 foot span, carried on 16-1/2" octagon pretensioned concrete piles. This structure was designed by Ray M. Murray, Consulting Engineer, Seattle, and the prestressed concrete was fabricated in the Concrete Engineering Company plant in Tacoma, and shipped 700 miles by barge to the job site.

This writer is in complete accord with Mr. Rundlett's suggestion for standardization.

Concrete Engineering Company has taken a step in this direction by tooling up with steel molds to produce the series of Standard I-beams shown in figure 6. These sections, all available on short notice, are cast with high-strength concrete, prestressed up to a working design stress of 3000 psi. in the concrete. With their high section-modulus to weight ratio, it is expected that they will find application in highway bridges of composite slab and beam design. They have been fabricated and delivered by truck to the job site in lengths up to 87 feet, and spans up to 100 feet now appear feasible and economical.



GIRDER

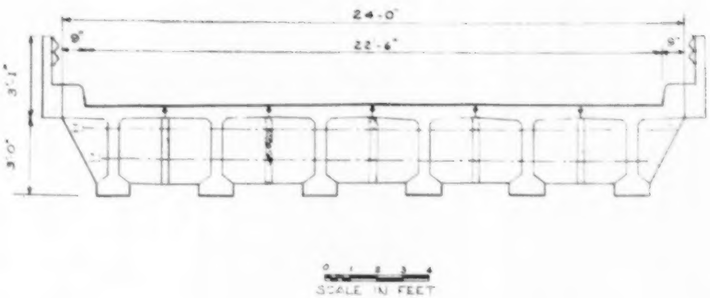
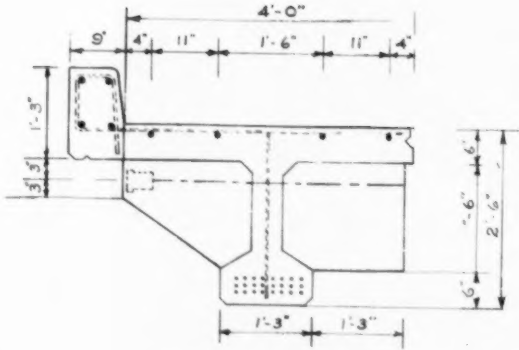


FIG. 1

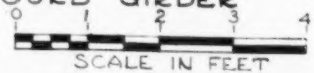
STANDARD HIGHWAY BRIDGE
FROM FACTORY-PRODUCED GIRDERS

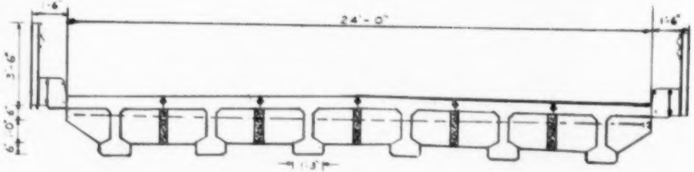
H20-S16 LOADING FOR
SPANS UP TO 90 FT.

(1)



CURB GIRDER





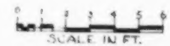


FIG. 2

STANDARD HIGHWAY BRIDGE
FROM FACTORY-PRODUCED GIRDERS

H 20-S16 LOADING FOR
SPANS UP TO 60 FT.

②

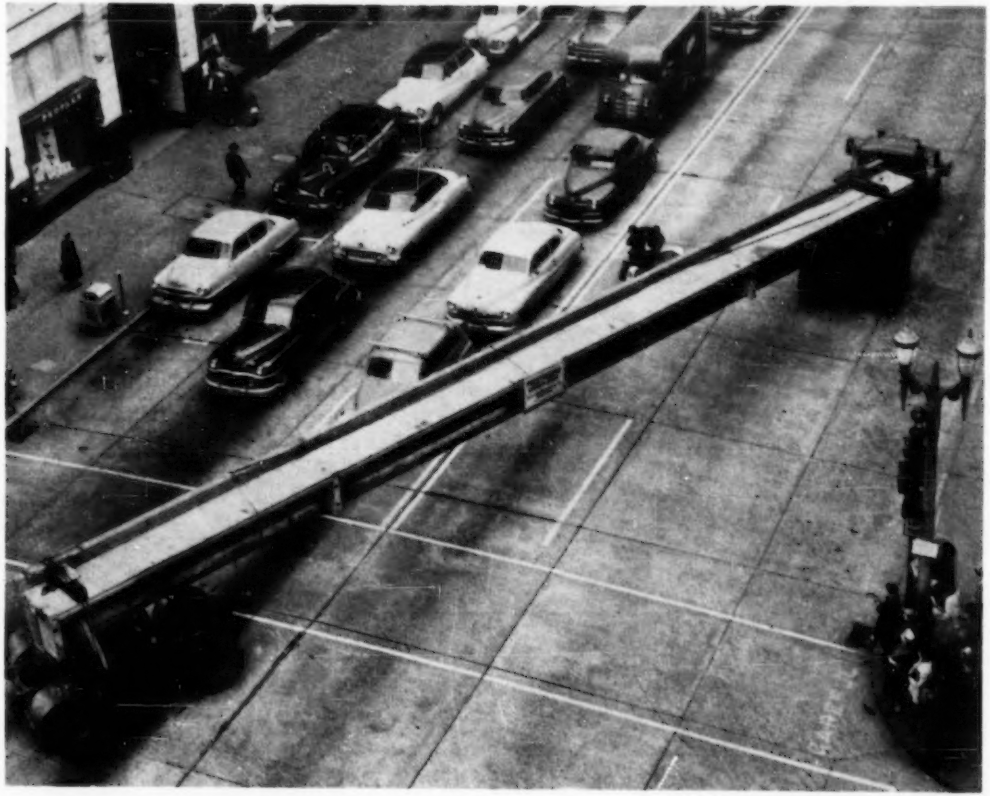


Fig. 3. Factory-produced pretensioned bridge girder 80 feet long delivered to job site 125 miles by truck and steering trailer.

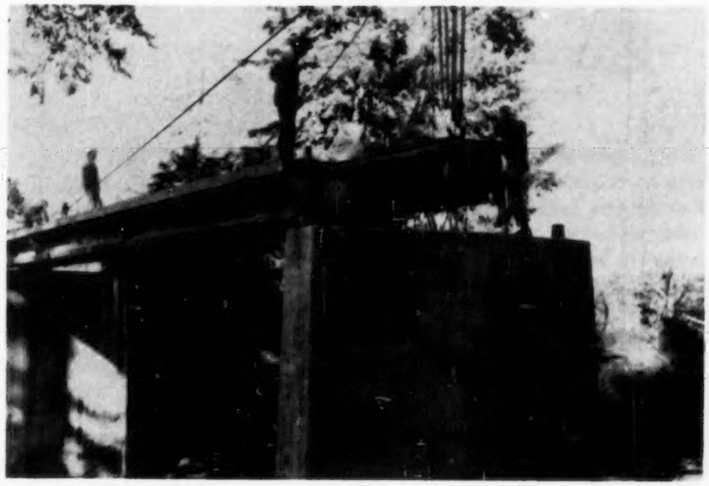


Fig. 4. Rainbow Falls State Park Bridge. Erection of 60-foot pretensioned girders with skyline.

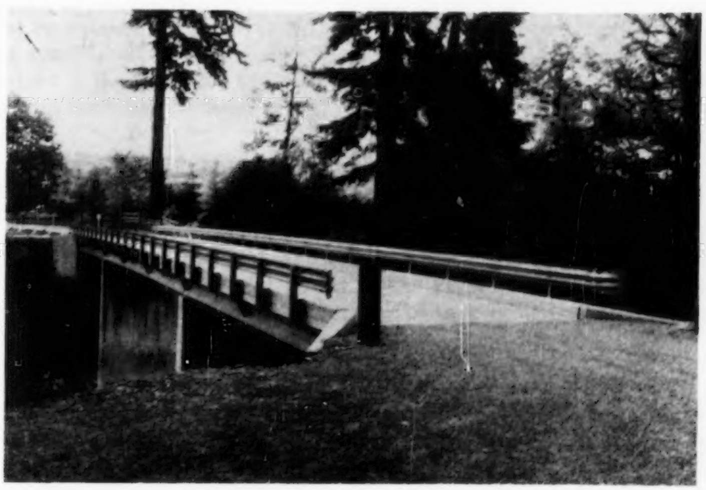
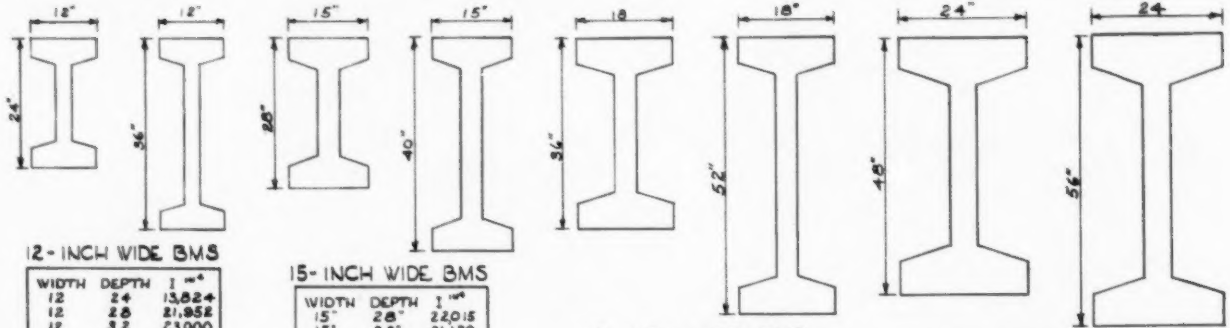


Fig. 5. Completed Rainbow Falls State Park Bridge.



12-INCH WIDE BMS

WIDTH	DEPTH	I in.⁴
12	24	13,824
12	28	21,952
12	32	23,000
12	36	31,020

15-INCH WIDE BMS

WIDTH	DEPTH	I in.⁴
15"	28"	22,015
15"	32"	31,120
15"	36"	42,100
15"	40"	55,130

18-INCH WIDE BMS

WIDTH	DEPTH	I in.⁴
18"	36	53,770
18"	40	70,260
18"	44	89,340
18"	48	111,300
18"	52	136,170

24-INCH WIDE BMS

WIDTH	DEPTH	I in.⁴
24"	48"	163,830
24"	52"	200,570
24"	56"	241,730

STANDARD FACTORY-PRODUCED I BEAM SECTIONS

FIG. 6

Discussion of
"INFLUENCE LINES FOR MOMENT AND SHEAR
IN A CONTINUOUS BEAM"

by Anthony Hoadley
(Proc. Paper 734)

BENJAMIN C. F. WEI,* J.M. ASCE.—The author presented a systematic method of analysing quantitatively the influence lines for moment and shear by the use of conjugate beam. The development of general equations for a 3-span continuous beam with constant moment of inertia in each span but varying from span to span is of particular interest and the author is commended for his good work. In the introduction the author pointed out that various influence line tables have been published and he named the tables by American Institute of Steel Construction and the "Influence Line Tables" by Griot-Lorsch. The writer would like to point out that the Influence Line Tables by Anger** gives far more complete information on influence lines than the two sources mentioned above and this book is extensively used by bridge engineers. This book gives influence lines in tabular form for moment, shear and reaction at tenth-point of a continuous 2-, 3- and 4- span beam of constant moment of enertia. The 2-span beam has ratio of spans ranging from 1:1 to 1:2.5 and the 3- and 4- spans deal primarily with symmetrical spans with the ratio of exterior span to interior span ranging from 1:1 to 1:2. The extensive coverage of influence lines in this book makes it seldom necessary for practical purposes to calculate the influence lines for continuous beam of constant moment of inertia and this fact is specially appreciated in design offices since the laborious work of finding influence lines is a great consumption of time and energy. It must be pointed out however that the formulas given by the author for the 3-span continuous beam are useful for finding generally the influence points at any points in the span, for treating unsymmetrical spans and for dealing with spans with constant moment of inertia in each span but varying from span to span.

In Table 3 of the paper, it is necessary to define the point $\underline{n}$ to be used in obtaining the values of A and H. Whereas the point $\underline{x}$ is clearly defined by the author as being measured from the end of that span furthest from the point P, the point $\underline{n}$ is actually measured from the intermediate support to point P in question. This is demonstrated by the fact that the author in his last example uses $\underline{n} = .40$ where the point P is 4 ft. to the right of support 2. It may also be observed in Table 3, for constant moment of inertia and $n = 0.2$ the value of A is found to be equal to infinity and no satisfactory value of influence line at that point can be obtained.

Although it is true that the equations set up by the author have the advantage that they could be evaluated by any computer, the computation of values of $\underline{y}$ by substituting a large number of $\underline{n}$ and $\underline{x}$ is still a long arithmatic procedure; furthermore the mechanics of computation is such that considerable care must be exercised in computing each individual point and the computer is unable to check the consistency of his values until all the individual values are one by one obtained. The writer prefers to use the method described below to obtain influ-

* Structural Designer, D. B. Steinmen, Cons. Eng., New York, N. Y.
** Anger "Zehnteilige Einflusslinien fur durchlaufende Trager" Band III 1949.

ence lines for moment and shear in a continuous beam. To start, it is advantageous to find first the influence line for moment over the intermediate support. After the influence line for the moment over the support is obtained influence lines for reactions, positive moments and maximum shears could be easily found by the use of statics. Fig. 1 (a) shows a 3-span continuous beam with constant moment of inertia. The continuities of the beam at a and b are broken to form simple statically determinate structures and unit moment Xa and Xb are applied successively at a and b. δ aa, δ ba, δ ab and δ bb denote the rotations at points a and b due to unit moment at a and b as defined in Fig. 1 (e) and (f). δ ap is the rotation at a due to unit load at P, P being any point in the beam. The equations for X_a and X_b due to unit load at P may be written as follows:

$$\delta ap + X_a \delta aa + X_b \delta ab = 0 \qquad (1)$$

$$\delta ap + X_a \delta ba + X_b \delta bb = 0 \qquad (2)$$

By solving the simultaneous equations and equating $\delta ab = \delta ba$, one obtains

$$X_a = \left(\frac{-\delta bb}{\delta aa\ \delta bb - \delta \overline{ab}^2} \right) \delta ap + \left(\frac{\delta ab}{\delta aa\ \delta bb - \delta \overline{ab}^2} \right) \delta bp \qquad (3)$$

$$X_b = \left(\frac{\delta ab}{\delta aa\ \delta bb - \delta \overline{ab}^2} \right) \delta ap + \left(\frac{-\delta aa}{\delta aa\ \delta bb - \delta \overline{ab}^2} \right) \delta bp \qquad (4)$$

By Maxwell's principle of reciprocal relation, the rotation at a due to unit load applied at P is equal to the deflection at P due to unit moment at a or $\delta ap = \delta pa$ and $\delta bp = \delta pb$. Hence Equations (3) and (4) become

$$X_a = \left(\frac{-\delta bb}{\delta aa\ \delta bb - \delta \overline{ab}^2} \right) \Delta pa + \left(\frac{\delta ab}{\delta aa\ \delta bb - \delta \overline{ab}^2} \right) \Delta pb \qquad (5)$$

$$X_b = \left(\frac{\delta}{\delta aa\ \delta bb - \delta \overline{ab}^2} \right) \Delta pa + \left(\frac{\delta}{\delta aa\ \delta bb - \delta \overline{ab}^2} \right) \Delta pb \qquad (6)$$

From Equations (5) and (6) it is obvious that to find influence line for the negative moment over support it is only necessary to apply unit moment at the supports a and b. From the unit moment diagram the rotations δ aa, δ bb, δ ab and δ ba are computed as the shear on the simply-supported beam with the moment diagram applied as load. The deflections Δpa and Δpb at the various points in the simple beam are found to be the moments at those corresponding points in the similar beam with the unit moment diagram applied as the load. Any number of ordinates for the influence line may be computed so long as equal number of points of deflections in the beam are found. In actual analysis, a systematic tabulation of the various quantities is suitable and results in minimum chance of error. It must be pointed out that, by using M/I diagram as load instead of M-diagram as explained above, Equations (5) and (6) can be used to find influence line for negative moment over the support for continuous beams with variable moment of inertia within each span (as in the case of girder with curved bottom flange) or varying from one span to another (as in the case of continuous stringer which has stiffer section at end span). For design of trussed structures elastic weights at various panel points are computed*

* See, for instance, D. B. Steinman's article "Continuous Bridge" in "Movable and Long-span Steel Bridges" by Hool and Kinnie.

and these weights are applied as loads to the hinged determinate system to obtain the rotations and deflections in Equations (5) and (6).

In design of continuous stringers or girders, bridge engineers are also interested in the maximum deflection in the beam under truck loading. The AASHO Specification **states that for steel beams or girders having simple or continuous spans the maximum deflection due to live load plus impact shall not exceed 1/800 of the span. After a section of continuous stringer is determined from shear and moment, it is necessary to check the extent of the live load deflection. An influence line for maximum deflection would be a valuable tool to determine the deflection in the beam under the moving load or loads. The curves shown in Fig. (2) are the result of analysis for the influence line for deflection in a 4-span continuous beam of equal span length and constant moment of inertia. Influence lines for deflections at 4.28 point in the exterior span and midpoint in the interior span are plotted. These points represent in general the points of maximum deflection. Note that in finding influence line for deflection at any point P it is only necessary to apply a unit load at point P and then to find deflections at all other points in the span due to this unit load. This is a direct application of the Maxwell principle of reciprocal relation. These curves offer a convenient check on live load deflection in continuous beam under wheel loads and result in great saving of time in practical design.

** "Standard Specifications for Highway Bridges" 1953 American Association of State Highway Officials, Washington, D. C.

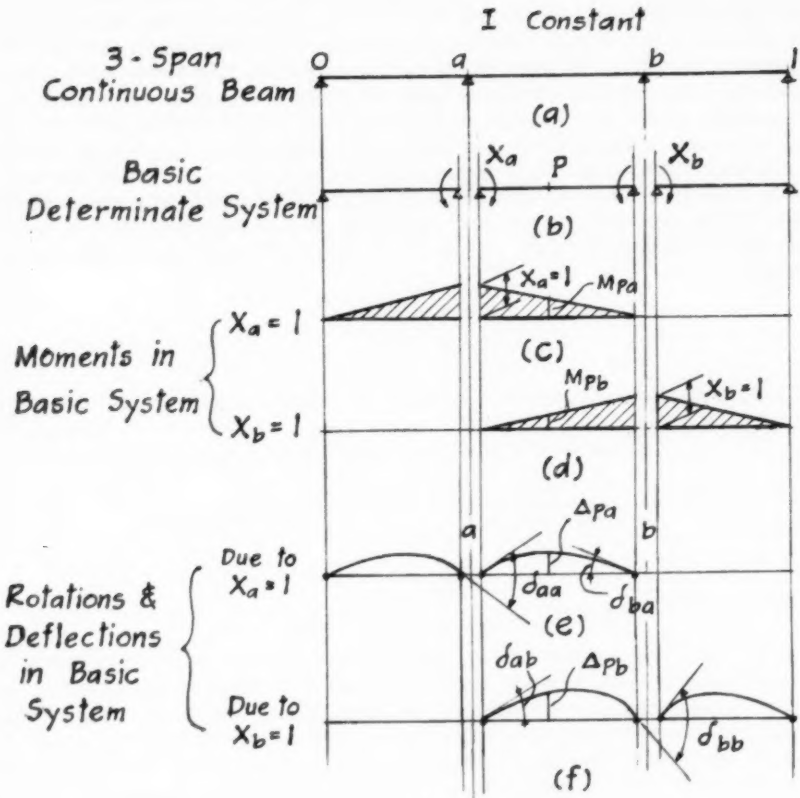
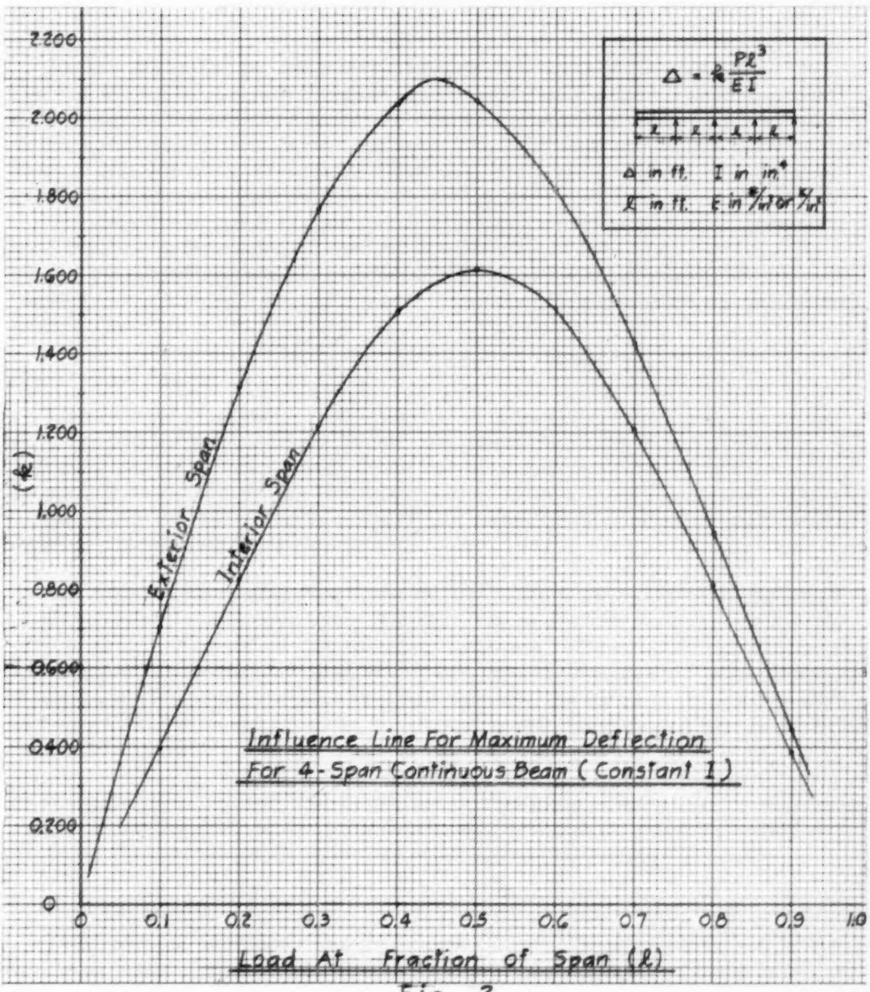

I Constant
3 - Span
Continuous Beam
0
a
b
1
(a)
Basic
Determinate System
X_a
P
X_b
(b)
$X_a = 1$
M_{Pa}
$X_a = 1$
Moments in
Basic System
(c)
M_{Pb}
$X_b = 1$
$X_b = 1$
(d)
Δ_{Pa}
a
b
Due to
$X_a = 1$
Rotations &
Deflections
in Basic
System
δ_{aa}
δ_{ba}
(e)
δ_{ab}
Δ_{Pb}
Due to
$X_b = 1$
δ_{bb}
(f)

Fig. 1



$\Delta = k \frac{P\ell^3}{EI}$
ℓ ℓ ℓ ℓ
Δ in ft. I in in.⁴
ℓ in ft. E in K/in² or K/ft²
Exterior Span
Interior Span
(k)
2.200
2.000
1.800
1.600
1.400
1.200
1.000
0.800
0.600
0.400
0.200
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
Influence Line For Maximum Deflection
For 4-Span Continuous Beam (Constant I)
Load At Fraction of Span (ℓ)

Fig. 2

Discussion of
"NATURAL FREQUENCIES OF CONTINUOUS FLEXURAL MEMBERS"

by A. S. Veletsos and N. M. Newmark
(Proc. Paper 735)

JACOB KAROL,[a] M. ASCE.—This paper is a notable contribution to the field of vibration analysis of beams and plane frameworks. The comprehensive tables of the physical constants required for analysis are of great value in the solution of a particular problem. The method of analysis should be readily understood by structural designers, since it is based on the familiar concepts of moment distribution.

The determination of the natural frequencies using the authors' procedure still involves a considerable amount of calculation. Alternate solutions for the three numerical examples will be presented which are simpler and more expeditious. The sign convention and nomenclature established by the authors will be retained.

Illustrative Example 1

This example involves the calculation of the natural frequencies of a four-span continuous beam. The authors' procedure requires the application of a unit rotation at the left support if the end is hinged or partially fixed, or a unit moment if the end is fixed. Rotations are computed at the successive supports by Eq. (10b), and for a hinged or partially fixed end at support $\underline{z}$, the moment must be determined from Eq. (15).

The alternate procedure involves the calculation of the effective stiffness at the successive supports using Eq. (3). The second term on the right-hand side of Eq. (3) represents the change in stiffness. The detailed calculations for example 1 are shown in Fig. K-1. The bottom line shows the effective stiffness at the supports. It should be noted that the value of $K_5 = 0.817\ EI/L$ represents the moment required to produce a unit rotation and agrees with the value determined by the authors.

If the end support had been fixed, the computation of the effective stiffnesses would be carried only to the penultimate support, and the natural frequency would be that for which the effective stiffness at that support was zero.

Illustrative Example 2

This example considers an open frame without sidesway which can be solved by the procedure used for continuous beams. The alternate solution is shown in in Fig. K-2. Since joint 1 is fixed, there is no change in stiffness at joint 2. Since joint 3 is fixed, the only change in stiffness at joint 4 is that involving member 2-4. The effective stiffness at $K_5 = 2.492\ \frac{E_1 I_1}{L_1}$.

Dividing the moment by the rotation computed by the authors for joint 5, $K_5 = 2.493\ \frac{E_1 I_1}{L_1}$.

[a] Prin. Design Engr., Howard, Needles, Tammen & Bergendoff, Kansas City, Mo.

Illustrative Example 3

This example involves a closed frame without sidesway. For this type of frame the authors present a general procedure requiring the solution of a second order determinant. The particular frame chosen for analysis is not as complicated as it may appear at first glance. Since it is only two stories high and since it is symmetrical about the center-line, the frame is basically a single quadrangular loop with branching members. Such a frame can be analyzed directly without the use of determinants.

The solution is shown in Fig. K-3 for $\lambda_z = 2.163$, which the authors indicate is the lowest natural frequency. In Fig. K-3b, the numbers above the line represent the stiffnesses and the numbers below the line the product of the stiffnesses and carry-over factors. The numbers at joints 3, 4 and 6 represent the total joint stiffness, except that the stiffness of members 1-3 and 2-4 have been modified to account for the hinged ends at 1 and 2.

For convenience, the loop 5-3-4-6 has been straightened out to simulate a continuous beam. It is first necessary to compute the effective stiffness at joint 4 using Eq. (3). Next a unit rotation is applied at the cut ends of joint 5. The rotations induced at joints 3 and 6 are computed using Eq. (13a). These rotations in turn produce rotations at joint 4. The rotation of joint 4 induces rotations at joints 3 and 6. The final rotations are shown in Fig. K-3c. An independent analysis for these rotations using simultaneous slope-deflection equations checked the results shown. Knowing the rotations at the joints, the moment at 5 is computed using Eq. (9). Since the moment is almost zero, the assumed frequency is very close to the natural frequency. The natural frequency determined by the writer corresponds to $\lambda_z = 2.164$.

An analysis for the lowest natural frequency assuming the columns fixed at joints 1 and 2 showed that it occurs at a value of $\lambda_z = 2.182$. This indicates that it may be possible to isolate and analyze the two top stories of a multistory building and obtain reliable results. The authors' comment regarding the limitations of such a procedure is invited.

It is worth noting that the solutions presented herein permit the recording of all the calculations directly on the structural diagram and make use of only three simple equations from the basic theory. All computations were made with a slide rule. The suggested procedure also saves considerable calculation. Designers may therefore find it preferable to the authors' procedure for problems where it is applicable.

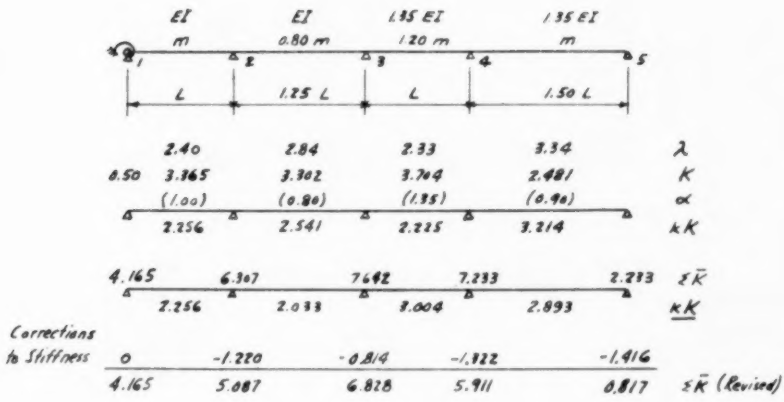


FIG. K-1 ILLUSTRATIVE EXAMPLE 1 - ALTERNATE SOLUTION

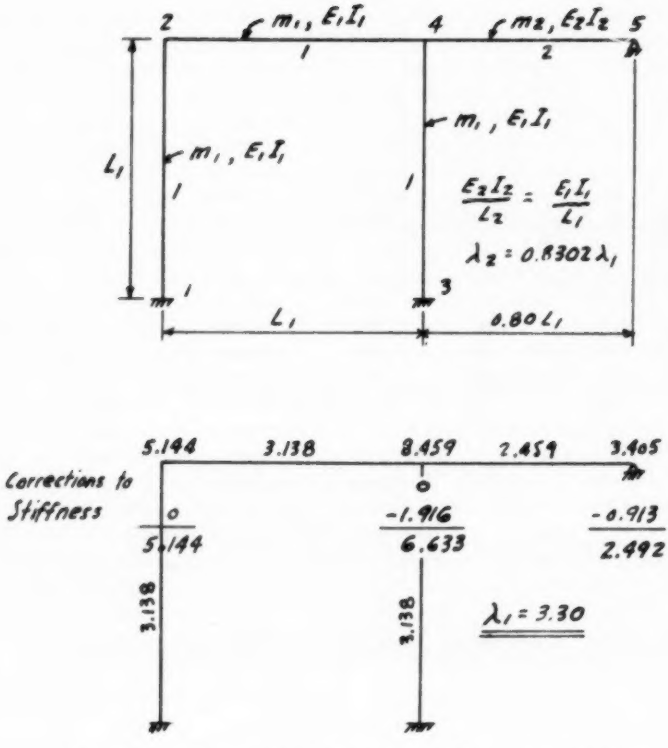
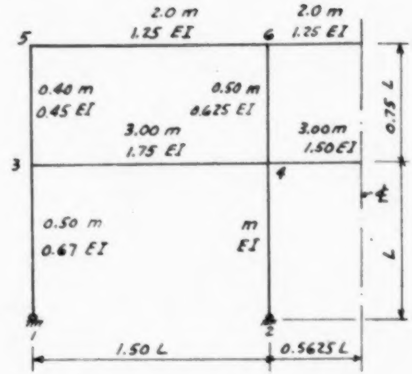
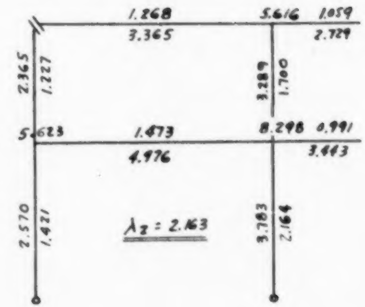


FIG. K-2 ILLUSTRATIVE EXAMPLE 2 - ALTERNATE SOLUTION



(a) Properties of Members



(b) Stiffness and Carry-Over Factors

Joint	5		3		4		6		5
$\bar{K}$	2.365		5.623		8.298		5.616		1.268
kK		1.227		4.976		1.700		3.365	
					-4.400 | -0.514				
Modified Stiffness at 4					3.384				
Rotations - Carried	1.00	→	-0.218	→	0.321 | 0.301	←	-0.599	←	1.00
Over			-0.549	←	0.622	→	-0.188		
Final Rotations	1.00		-0.767		0.622		-0.787		1.00

(c) Joint Rotations For Closed Loop 5346

$$M_S = (2.365 - 1.227 \times 0.767 + 1.268 - 3.365 \times 0.787)\frac{EI}{L} = 0.042\frac{EI}{L}$$

FIG. K-3 ILLUSTRATIVE EXAMPLE 3 - ALTERNATE SOLUTION